Pressure Distribution Modelling in Cake Expression

NDEKWU O. B. and ADEMILUYI J. O.
Department of Civil Engineering, Faculty of Engineering, University of Nigeria, Nsukka
Corresponding Author: NDEKWU O. B.
beneathworld@yahoo.com

ABSTRACT
The principle underlying expression basically involves the separation of a solid from the liquid in which it is suspended by passing the mixture through a porous medium with pore sizes too small to allow the passage of the solid particles at an applied pressure. The analytical method of the finite element method has been used to determine pressure distribution along sludge cake height in a filter press expression process. The finite element formulation basically involves the properties of the sludge in question as distinguished from other solution. The solution domain was idealized as a one-dimensional quadratic shape function for the purpose of this analysis and the displacement function formulation method were employed in solving the pressure distribution. The ranges of displacement in a sludge cake height with increase time of expression increases with a decrease in sludge depth. The displacement increases continuously with increase time of expression. It was discovered that pressure increases with decreasing height of the cake at different time of expression. It was also discovered that pressure increases with expression time.

Key words: Expression, Filter press, Pressure distribution, Displacement distribution, Sludge cake.

INTRODUCTION

Sludge which is produced as a by-product of all treatment processes have considerable application potentials (such as fertilizer and soil conditioners).The liquid sludge contains about 90-98% water and sometimes it is transported long distance for ultimate disposal. Economically, it is very important to reduce the sludge volume by removing as much water as possible to ease the transportation, handling and disposal cost. Current research shows that because of the high water content in sludge which is about 97.5%, sludge generated is difficult to handle (3). The resurgence of interest in separation technology in recent years has led to a revival of cake filtration studies. However, the improvement of this technology relies on understanding with greater insight and better information of the various aspects of the filtration process (21).
The handling and disposal of this sludge is one of the greatest challenges facing the environmental engineer. The sludge has high water content and compressibility attribute and as such it is expedient to dewater it to reduce its volume and prevent environmental health hazard. The dewatering of sludge using the filter press method has been in existence far back as in the 1920s. Since then, a number of equations have been presented by various contributors aimed at improving the

performance of the sludge filtration/expression process, (8, 16, 12, 3, 2, 15, 5, 1). However, their research was limited to experimental work which could not provide an insight into the interactive nature of sludge filterability. As previously sighted in literature (5, 1) the inapplicability of Darcy's law of fluid flow to the dewatering process stems from the fact that Darcy's law is only applicable to rigid materials where porosity is constant. This uniformity in porosity implies a corresponding uniformity in pressure, hence a uniform correspondence in displacement throughout the cake height. This is not true with compressible or deformable materials in for sludge cake. The solid-like network structure can support stress elastically up to a given effective pressure, defined as the compressive yield stress (7). Further, the network structure collapses irreversibly at effective pressures above due to breaking and rearrangement of bounds between the particles and/or formation of more contact points between the particles (14).This is not true with compressible or deformable materials in for sludge cake. Porosity decreases from sludge to the cake height closest to the septum (17, 6, 18, 19, 10, 11). In the course of this porosity variation, pressure also varies. Even though the direction of porosity and pressure variation will be intuitively assessed, there is still the need to provide a theoretical base for such assessment. It is in the light of the foregoing that finite element method of analysis has been used in this research to evaluate the distribution of pressure along the expressed cake height. It is hoped that the study will find practical application in the evaluation of models necessary to describe cake expression phenomenon.

FINITE ELEMENT MODEL FORMULATION

During the filtration process, the sludge material deforms with time to form a sludge cake, while the liquid or water in the pores gradually squeezes or diffuses out. This phenomenon has been approximated to occur in one dimension for our solution. The finite element matrix is hinged on the formation of a stiffness matrix which contains only the sludge properties. The analysis was based on only one element, before the assemblage or global element was built. The following assumptions were made and the required stiffness matrix was formulated by a series of steps.

1. The medium is homogeneous in terms of constituents.
2. The sludge chemical properties are constant throughout the expression cycle. This is true since no chemical reaction takes place during the expression cycle.
3. At the start of the filtration process, because the medium is saturated at $t<0$, the fluid carries all the applied load (pressure) and the initial condition are $u(y, 0)$ = applied pressure and $\sigma'(y, 0) = O$ where O is effective stress.

As time elapses, the pore pressure dissipates and the load is gradually transferred to the sludge skeleton and σ' increases (20). Considering this increment of σ' with time, it is assumed for this work that u_0 (initial pressure before load) is small, therefore the governing equation

$$\sigma = \sigma' + u_0$$

is approximated as,

$$\sigma = \sigma'$$

after the formation of the sludge cake.

4. It is assumed that under constant external load, total stress does not change during expression.

For axial deformation of a column element, the stereo displacement relation, assuming very small strain is given as;

$$\varepsilon_y = \frac{du}{dy} \tag{1}$$

For quadratic shape functions

$$\frac{du}{dy} = \frac{1}{L}\left[\left(-1 - {}^{4x}/_{L}\right)4\left(1 - \frac{2x}{L}\right)\left(-1 + \frac{4x}{L}\right)\right]\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{2}$$

$$x\ is\ either = 0, {}^{-L}/_{2}\ or\ {}^{L}/_{2}$$

On assumption that x = 0, where N_1=1, and N_2 =N_3 =0; confirms the accuracy of the shape function. Then,

$$\frac{du}{dy} = \frac{1}{L}\begin{bmatrix} -1 & 4 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3}$$

Where ε_y = axial strain, the rate of change of deformation:

u = displacement, and

y = space.

The relationship between action and response with respect to the applied pressure and axial deformation is the stress- strain of the material expressed.

$\sigma_y = E\ (\varepsilon_y - \varepsilon_0)$--------------- (Hooke's law)

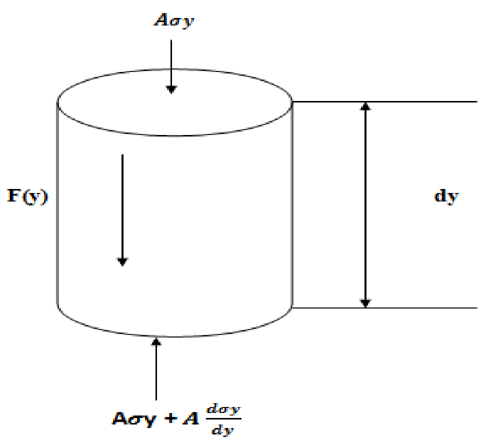

Figure 1: Typical Diagram of Active Forces in an Expressed Material

From the column element (Figure 3.2) above, the governing differential equation for the case of axial deformation may be derived based on the concept of equilibrium of forces for on elemental volume of the column. Hence;

$$A\sigma_y - \left(A\sigma_y + \frac{Ad\sigma y}{dy}\right) + f(y) = 0$$

$$A\frac{d\sigma_y}{dy} - f(y) = 0$$

Where:

A = cross-sectional area of the column:

σ_y = stress abuy y – axis

F(y) = body force (weight), surface loading (fraction) or applied force ${du}/{dy}$ at local points.

$$A\frac{dE(\varepsilon_y - \varepsilon_0)}{dy} - f(y) = 0 \qquad AE\frac{d^2u}{dy^2} - AE\frac{d\varepsilon_0}{dy} = f(y)$$

Letting initial strain term $\varepsilon_0 = 0$

$$AE\frac{d^2u}{dy^2} - f = 0$$

This is one dimensional second order differential equation, used as the basis for illustrating axial deformational of cake expressed. Using Galerkin's finite (weighted residual method). We have:

$$\int_e^{N^T} \left(AE \frac{d^2u}{dy^2} - f \right) dy = 0 \tag{4}$$

The second order term in equation (4) is reduced to the first order equivalent via integration by parts, thus, we have;

$$\int_0^L N^T \frac{du}{\partial y} - \int_0^L AE \frac{dN^T}{dy} \frac{du}{dy} dy - \int N^T f dy = 0 \tag{5}$$

$$\int_0^L AE \frac{dN^T}{dy} \frac{du}{dy} dy + \int N^T f dy = 0 \tag{6}$$

Derivation of element equation with a quadratic shape function

(Term 1)

$$\int_0^L AE \frac{dN^T}{dy} \frac{du}{dy}\, dy = \int_0^L AE\, \frac{dN^T}{dy} \frac{d}{dy} ([N]\{u\}) dy$$

$$= \int_0^L \frac{AE}{L^2} \begin{bmatrix} -1 - \frac{4y}{L} \\ 4\left(1 - {}^{2y}/_{L}\right) \\ \left(-1 + {}^{4y}/_{L}\right) \end{bmatrix} \left[\left(-1 - {}^{4y}/_{L}\right) 4 \left(1 - {}^{2y}/_{L}\right) \left(-1 + {}^{4y}/_{L}\right) \right] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} dy$$

$$= \frac{AE}{3L} \begin{bmatrix} 7 & 8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{7}$$

Term 2

$$\int_0^L N^T f dy = \int_0^L f \begin{bmatrix} \left(1 + {}^{y}/_{L}\right) \\ \frac{4y}{L}\left(1 - {}^{y}/_{L}\right) \\ -{}^{y}/_{L}\left(1 - {}^{2y}/_{L}\right) \end{bmatrix} dy = {}^{fl}/_{6} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \tag{8}$$

Assembling term 1 and 2

$$= \frac{AE}{3L} \begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{fL}{6} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \tag{9}$$

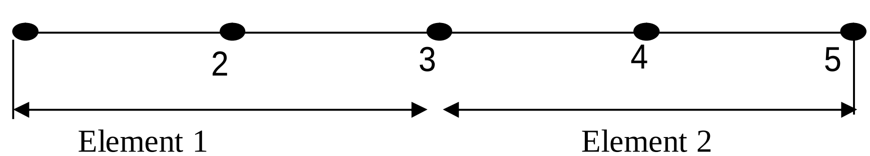

Figure 2: Quadratic finite element discritization option

On globalization, the assemble equation becomes

$$\frac{AE}{3L}\begin{bmatrix} 7 & -8 & 1 & 0 & 0 \\ -8 & 16 & -8 & 0 & 0 \\ 1 & -8 & 14 & -8 & 1 \\ 0 & 0 & -8 & 16 & -8 \\ 0 & 0 & 1 & -8 & 7 \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \frac{FL}{6}\begin{bmatrix} 1 \\ 4 \\ 2 \\ 4 \\ 1 \end{bmatrix} \quad (10)$$

Equation (10) may be taken as global finite element equations given that field variable (displacement, U_i, i =1, 2, 3, 4, & 5 for which solution is sought is expressed in global nodes).

Observing the stiffness matrix obtained in equation (10) the only parameter that depends on the sludge properties is the Elastic modulus. The cross-sectional area depends on the expression medium used. The sludge under consideration is a viscous when it is a mixture of solid and liquid. In this work a relationship between elastic modulus and viscosity was obtained considering a dimensionless constant, k.

By dimensional analysis, the fundamental unit of elastic modulus is given as;

$$E = \frac{N}{mm^2} = \frac{M}{LT^2} \quad (11)$$

the fundamental unit of viscosity is

$$\mu = kgm^{-1}s^{-1} = \frac{M}{LT} \quad (12)$$

Comparing both units, the relationship sought becomes

$$E = {}^{K\mu}/_{t} \quad (13)$$

The strain ($\epsilon = \Delta V/V$) varies with time (t) at constant pressure as the expression process continues.

Rearranging equation (13) as,

$${}^{P}/_{\epsilon} = {}^{K\mu}/_{t} \quad (14)$$

Rearrange equation (14) as

$$P/_{K\mu} = \epsilon/_{t} \quad (15)$$

Therefore, ϵ/t =constant = S. A graph of ϵ versus t gives a slope "S" (either linear, quadratic or power function slope). The unit of S is (sec^{-1}).

Thus,

$$\frac{P}{K\mu} = S \quad (17)$$

Therefore the required dimensionless constant is:

$$K = P/_{S\mu} \quad (18)$$

The values of K can now be substituted in equation (3.40) to obtain the replacement for elastic modulus in the stiffness matrix. Thus,

$$E = P/_{St} \quad (19)$$

Since the global matrix depends on the thickness of the cake formed at the time of consideration, it becomes necessary to determine the thickness of the cake before discritization, minimization and assemblage. Thus, a time dependent relationship is required to obtain the thickness of the cake. In 1981, Corapciogh (13) derived a time dependent expression using conservation of mass for cake thickness (L) as,

$$L = \frac{1}{120 + t}\left[[1.9563t + 0.4446\, In(0.02 + 0.000167t)] - \Big|_{0}^{t} + 1.7393\right] \quad (20)$$

Where t is in secs and L in cm.

MATERIALS AND METHODS

To solve equation (10), values of A, E, L and F are needed for each expression time. This is because the number of element depend on the thickness of the cake, but a constant length (L) is to be used for convenience. The total cake thickness of any expression time was calculated using equation (20) before discritization, which means that the number of element used for the analysis increases with expression time. In calculating the maximum cake thickness obtainable at expression times of 10mins, 20mins, 30mins and 70mins, equation (20) was used. Sludge material (cassava mesh) was gotten from an agro farm at the University of Port Harcourt, Choba, Port Harcourt, Rivers State of Nigeria. Ten minutes was considered well enough for a cake to be formed and a constant filter pressure of 4.22kg/cm^2 (60Psi) was used with gas deliquoring filter press equipment. The data was only used to calculate the strain and the elastic modulus at different time during the expression process. The polynomial slope 'S' was calculated by plotting strain ($\in_y$) versus time (t) using equation (15), to permit the evaluation of 'S'. Since S, t and P are known, equation (xxx) was used to evaluate the elastic modulus at any point along the cake height. For all finite element analyses carried out at different expression times, the area of filter paper used was 50.272 cm^2. A constant element length of 0.2cm was used for convenience of all finite element analysis carried out at all the expression times tested. Also for a particular time of expression, constant elastic modulus E was used. Body force F of the medium was found to be 0.001kg/cm^3. Substituting these values in equation (10), and solving would give values of U_1, U_2, U_3, U_4, U_5, U_6 and U_7. These values were then substituted in equation (3) to permit the evaluation of strain ($\in_y$) at the nodal points. Equation (3a) was then used to evaluate σ_y. This procedure was repeated for all elements generated at expression times of 10mins, 20mins, 30mins, 40mins, 50mins, 60mins and 70mins.

RESULTS AND DISCUSSION

Displacement as a trend in cake expression shows the depth variation along the cake height as an evidence to prove the effect of porosity variation in a typical sludge expression process. Results from finite element model as shown in Figure (3, 4) displays that there is a continuous increase in displacement at constant pressure with increase time along the expressed cake. This is due to the continuous dissipation of excess water (filtrate) from the pore spaces of the filtered cake. Hence, there is a continuous axial deformation along the expressed cake at varying heights per time (minutes). As shown in figure (5) the quadratic model proves to be more effective in describing the pattern of sludge cake height distribution at a constant pressure. Continuous increase in displacement from the start of the expression at 10 minutes to 70 minutes shows that, though filtrate volume may reduce per time of consideration, consolidation of the filtered cake continues. Hence, displacement increases per unit time along the cake height as time of filtration increases.

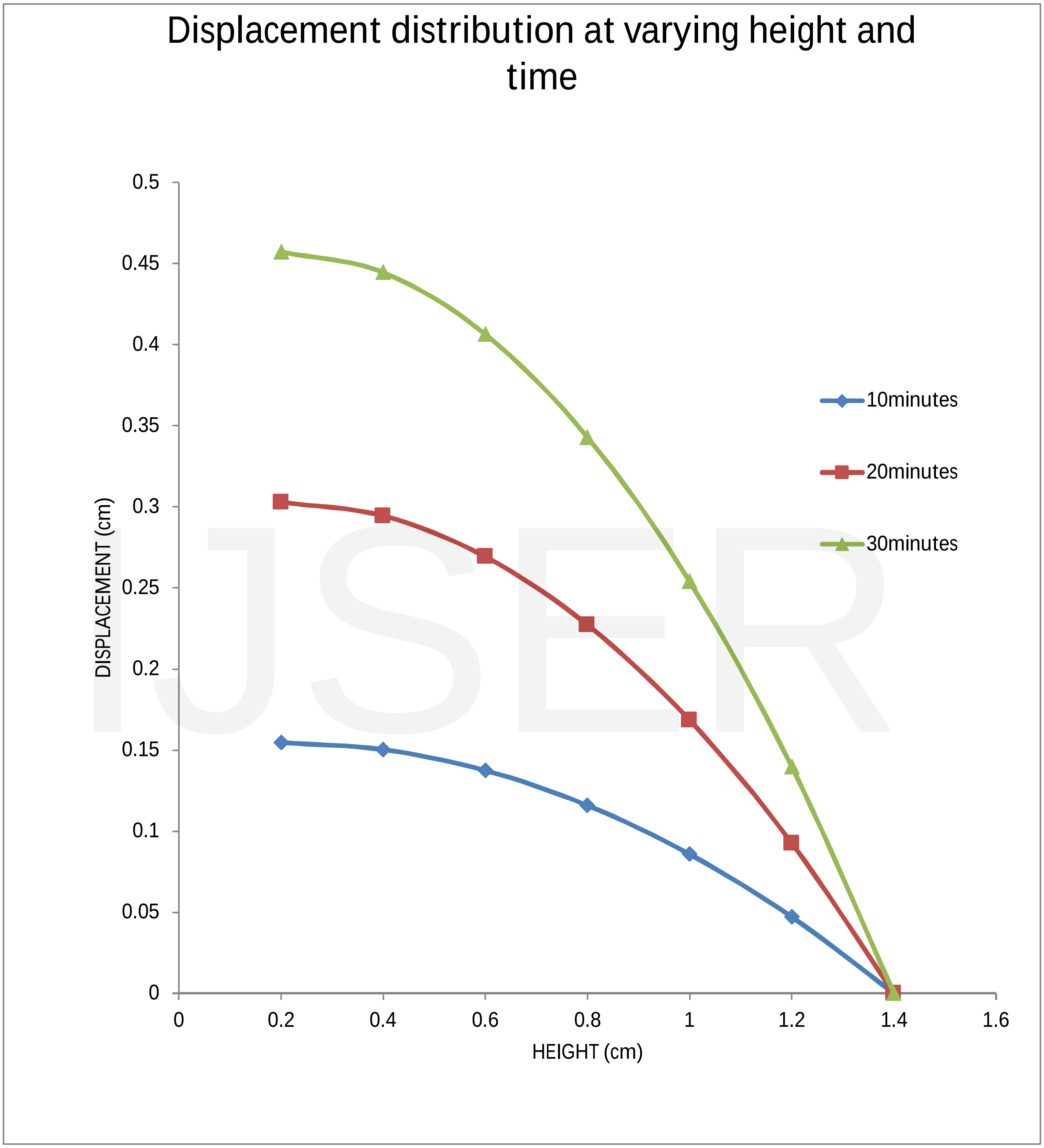

Figure 3: Quadratic Model Displacement Distribution at Varying Heights for 10, 20 And 30 Minutes.

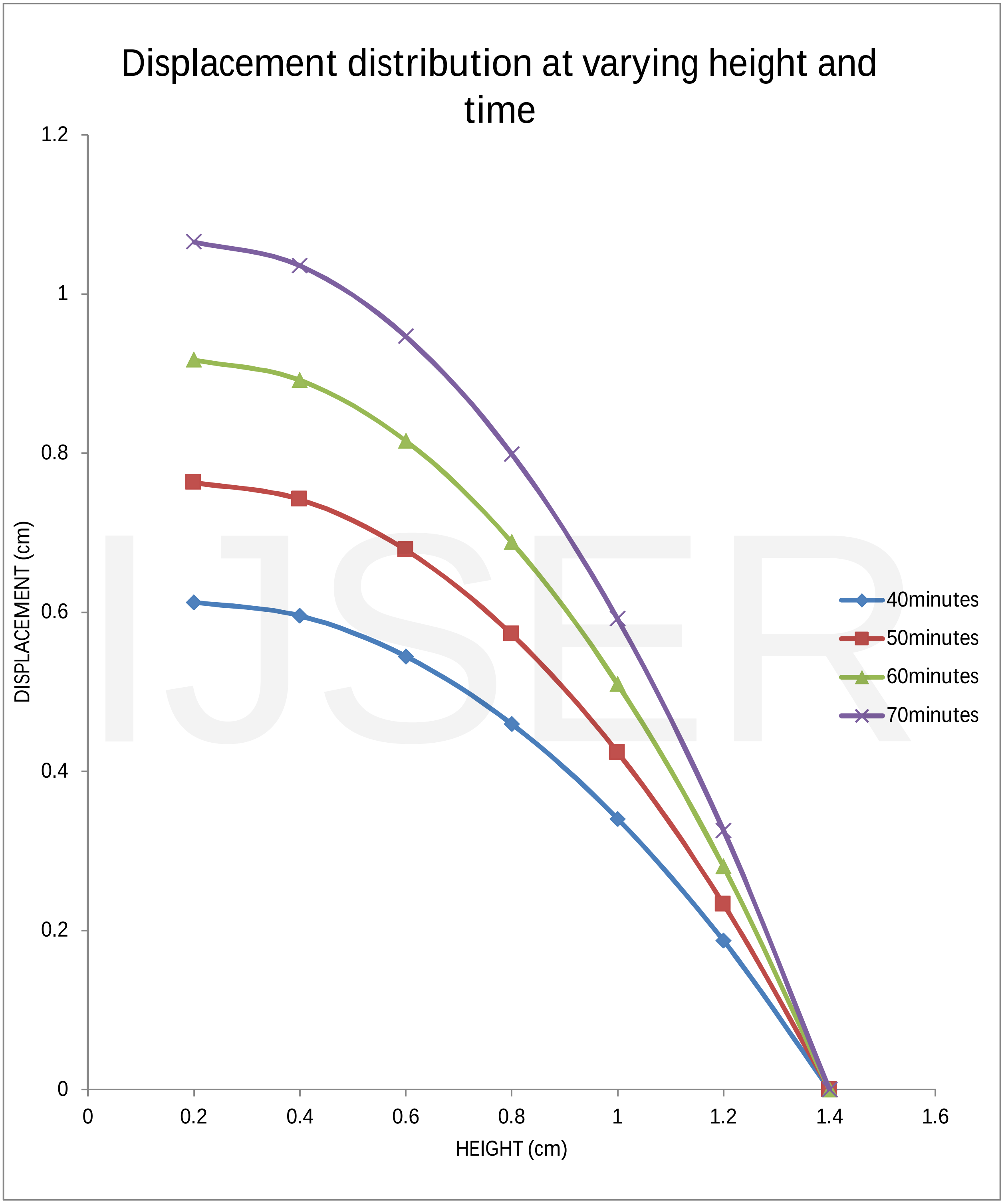

Figure 4: Quadratic Model Displacement Distribution at Varying Heights for 40, 50, 60 and 70 minutes.

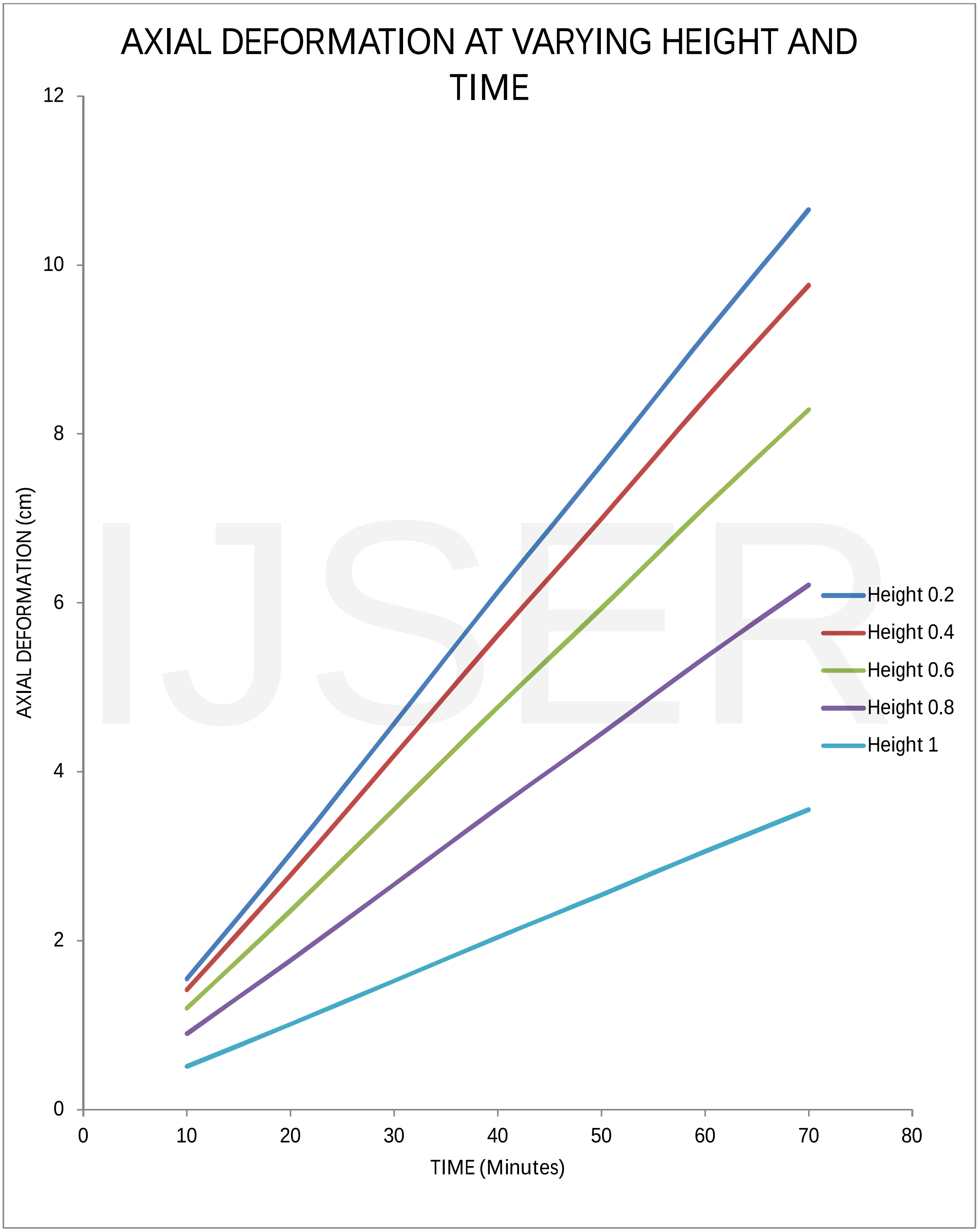

Figure 5: Quadratic Model Axial Deformation at varying height and time

Pressure σ_y at various nodal points for expression times of 10mins, 20mins, 30mins, 40mins, 50mins, 60mins and 70mins are as shown in figures (6, 7 and 8). Figure (9) is the plot of pressure drop versus cake height at various expression times.

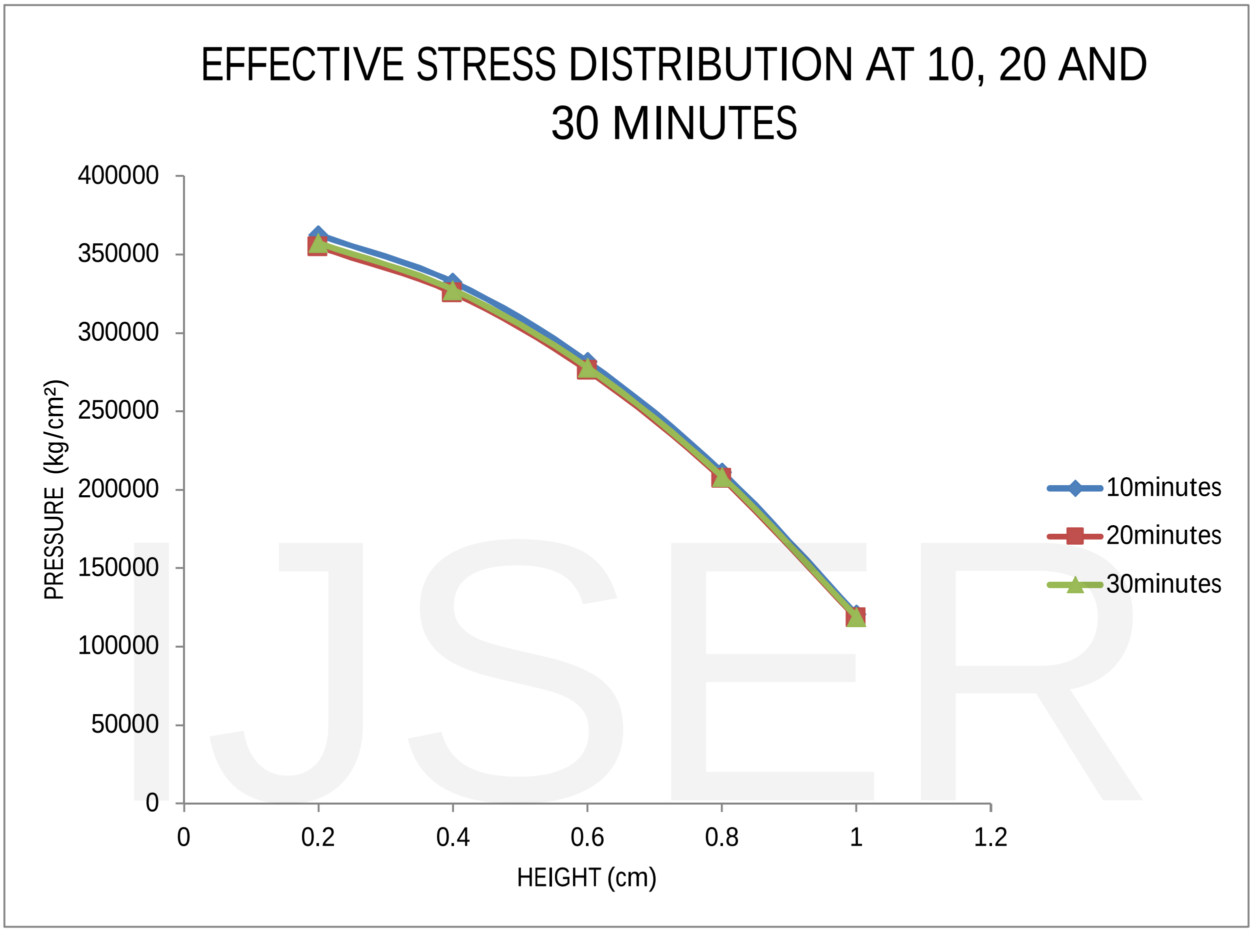

Figure 6: Effective stress (pressure) distribution at 10, 20 and 30 minutes.

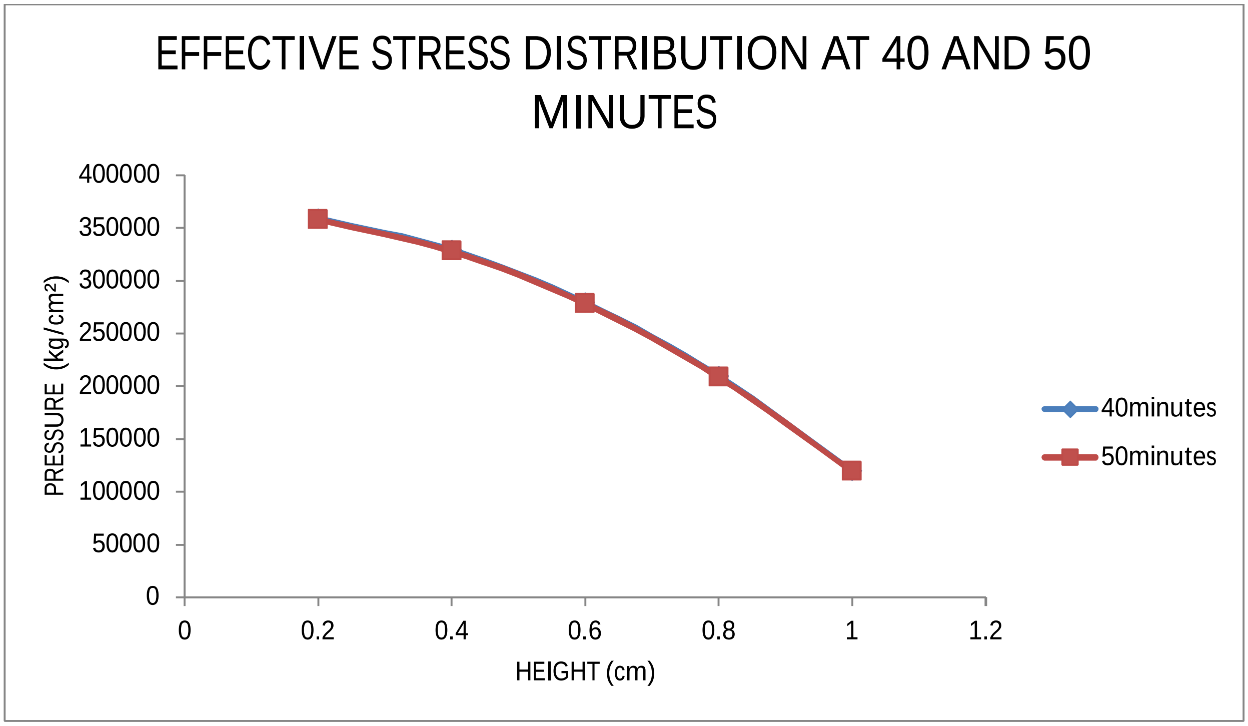

Figure 7: Effective stress (pressure) distribution at 40 and 50 minutes.

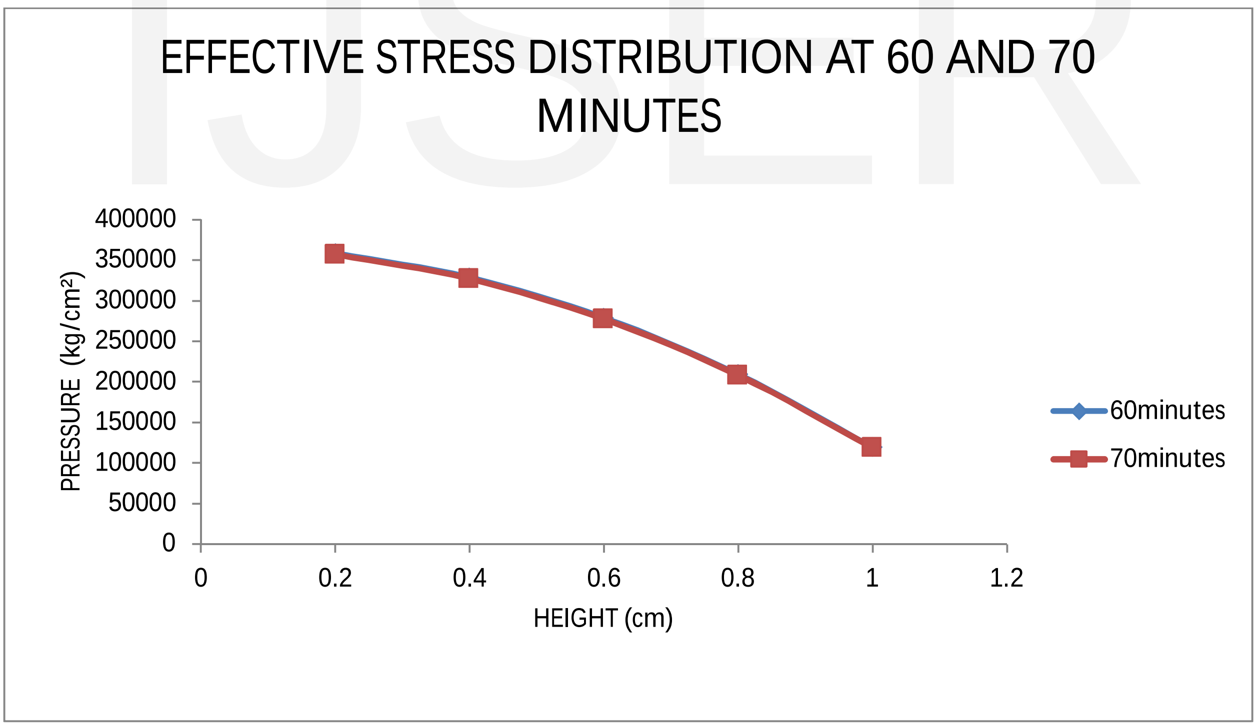

Figure 8: Effective stress (pressure) distribution at 60 and 70 minutes.

The problem of determination of pressure distribution along sludge cake can be likened to the secondary compression in consolidation theory in soil. In the Terzaghi theory (20), a change in void ratio is due entirely to a change in the effective pressure brought about by the dissipation of excess pore water pressure, with permeability along governing the time dependency of the process. However, experimental results has shown that compression does not cease when the excess pore pressure has dissipated to zero but continues at a gradually decreasing rate under constant effective stress. It is this slow continued compression that continues after the excess pore pressures have substantially dissipated that is called secondary compression. Ideally there must be small excess pore pressure during secondary compression to cause water to flow from the cake.

Results obtained using finite element analysis in this research study has proven that the effective stress increases with decreasing height of sludge cake at different time of expression. The filter cake has an unstable structure, stabilized by some forces related to the flow velocity. In general an extremely loose packing exists and disturbances will easily alter the structural arrangement. One of such disturbances is the unavoidable result of the flow process. The drag force exerted by the liquid on the particles and on the filter material causes a cake pressure or filter medium pressure, which increases with the direction of flow.

The reduction in porosity of the cake can be in two forms – a reversible and irreversible process. In the reversible process (also called compression) the particles retain places of contact. In such a case swelling of the cake will occur after flow has stopped. The irreversible process in which the particles are displaced with respect to each other is also called consolidation. It is only by this last process that the cake moves to its stable state which is the most densed packing possible.

Hence, to supplement the forgoing general increase of pressure with decreasing cake height is the corresponding effective decrease in the applied filter pressure with increasing cake height. It is reasonable to suggest that the applied filter pressure is most effective at the cake layer closest to the septum and least effective at the sludge cake interface.

Plots of effective pressure distribution has shown to be uniformly increasing with decreasing cake height in this quadratic model formulation, but cases might arise where opposite forces are developed as opposed to the negative compression suction pressure produced in some filtration processes(4). This condition may be caused by the formation of crack in the sludge cake at various sections along the cake height. This is because of the anisotropic nature of the filter medium, since the openings or pores are not uniform in size and are not distributed uniformly over the surfaces (which eventually leads to anisotropy of the initial layers of the cake), consolidation may not take place simultaneously over the whole cake. Thus at the outset the denser parts of the filter bed are consolidated at the cost of the less dense parts resulting in “crack” formation. However, this phenomenon may not be very distinctive in sludge cakes with small thickness.

Also a plot of pressure drop at particular times of expression versus cake height (Figure 6) indicates that pressure increases with time.

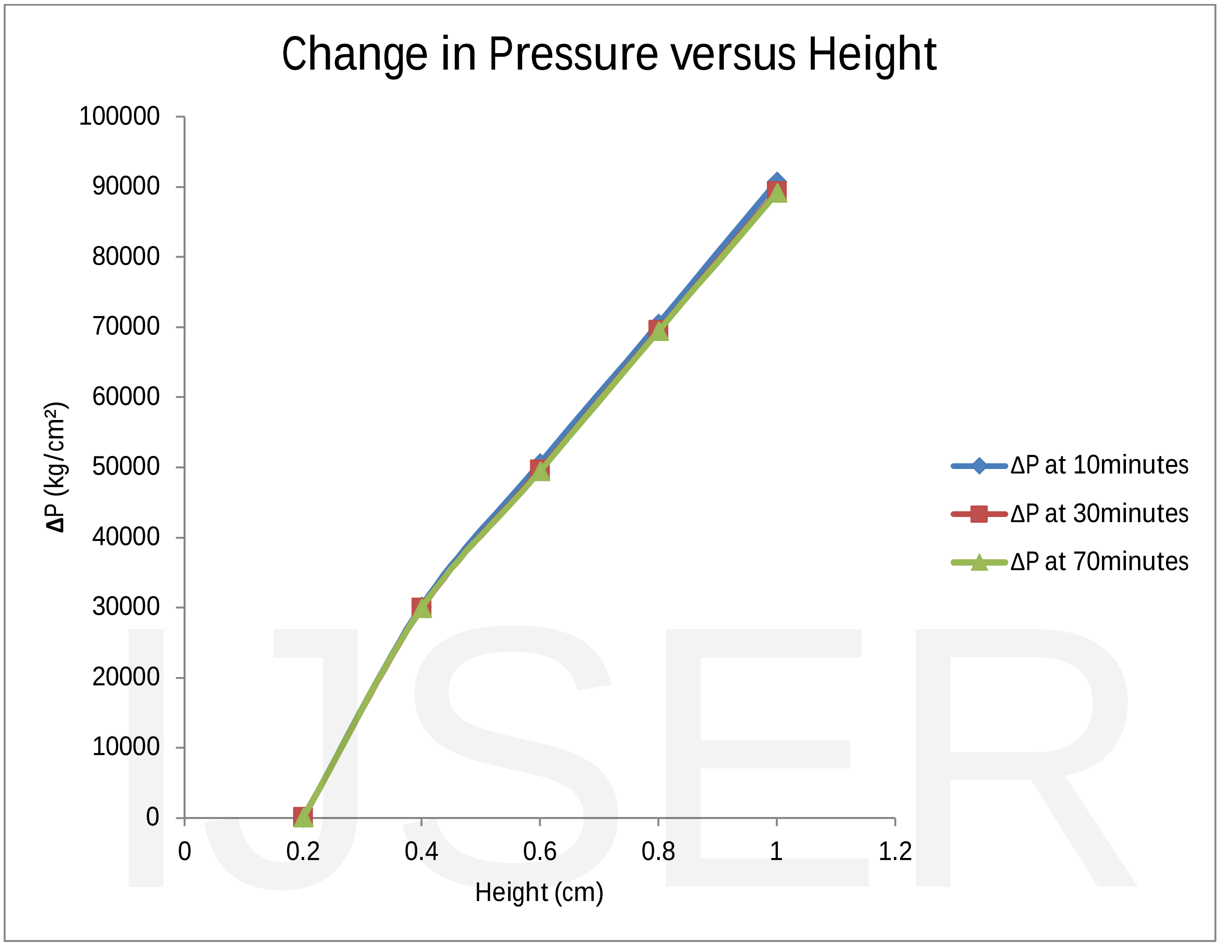

Figure 9: Schematic Pressure drop increase with Cake Height

This may be attributed to the continued dissipation of excess pore water pressure with increasing expression time.

Sludge filterability is a function of porosity which in turns depends on the pressure difference across the filter cake. Therefore the study of pressure distribution along the height of a sludge cake during expression can aid in the variation of the filterability parameter used to quantify sludge dewatering.

CONCLUSIONS

The following conclusions have been drawn from this research study.

1) The assumption of constant average specific cake resistance independent of cake thickness has been proven wrong for constant-pressure expression. The specific cake resistance of filter cakes in constant-rate expression, however, is not just affected by the solid

concentration but also the rate of flow of filtrate (which determines the extent of cake displacement (cake thickness) with time and effective pressure)

2) Displacement along the sludge cake height increases with an increase in the effective pressure.
3) Effective stress increases with decreasing height along sludge cake at different expression time. This has been explained as being due to drag force exerted by the liquid on the particles which increase with decreasing height. However, this could also be due to the distribution of the effective filter press pressure along the cake height in which case, the filter press pressure is most effective at cake layers closest to the septum and least effective along sludge-cake interface.
4) The increase in pressure with filtration time noted in this research work has been attributed to the continuous dissipation of excess pore water pressure with increasing filtration time.
5) It has been demonstrated that the expression mechanisms under constant pressure consist of two flow phenomenon; of filtration and consolidation, and these phenomena, though occurring progressively from filtration, changes at a transition stage with increased time of filtration, with no visible flow of filtrate, but with evidences of displacement in the sludge cake height as a result of solid settlement. A process called consolidation.

NOTATIONS

P = Pressure
μ = Coefficient of viscosity

u_0 = Pore pressure
O = Effective stress
L = Element length
U = Displacement
A = Cross sectional area
[B]=Strain – nodal displacement transformation matrix
σ_y = Stress in y-direction
$\in_y$ = Strain in y-direction
$[K^e]$ = Element stiffness matrix
$\in_y$ =Strain vector
[C]=Stress-strain matrix
$[R^e]$ = Element load vector
$[d^e]$ = Element nodal displacement vector
E = Elastic modulus
F = Body force
Ơ = Applied nodal loads
K = Dimensionless constant
S = Slope of strain versus time plot
T = Time
V = Volume of filtrate

REFERENCES

1) Ademiluyi JO (1986). Development in the constant Vacuum cake filtration theory: Proceedings of the Engineering section of science association of Nigeria Vol. 6 & 7.
2) Ademiluyi JO, Egbuniwe N and Agunwamba JC (1987). A dimensionless number as an index of sludge dewaterability, Journal of Engineering for Development, Vol. 1, p. 1-12.
3) Ademiluyi JO, Anazodo UG and Egbuniwe N (1983). Filterability and compressibility of sludge pt 11. Effluent and water treatment journal vol. 23, no 1 pp 25-30.
4) Ademiluyi JO and Onyemelukwe OU (1988). Concentration distribution in sludge filtration. African Journal of Science and Technology,ANSTI Vol 7. No. 1
5) Anazodo UG (1974). Dimensional equation for sludge filtration, effluent and water treatment journal vol, 14 n0 9 pp517-523.
6) Bear J and Bachmat Y (1991), Introduction to Modeling of Transport Phenomena in Porous Media, Kluwer Academic Publishers.
7) Buscall R and White LR (1987). The consolidation of concentrated suspensions- the theory of sedimentation, J. Chemical Society, Faraday Transactions **1**, 873-891.
8) Carman PC (1934). A Study of Mechanism of Filtration Part II, Journal of the Society of Chemical Industry, Transactions Communication. 53(6):1591-1651.
9) Carman PC (1938). Fundamental Principles of Industrial Filtration. Transactions – Institution of Chemical Engineers; 16: 168-188.
10) Chellappah K Tarleton ES and Wakeman RJ (2009). Cake filtration and sedimentation of fibre/particle mixtures, Poster session presented at: 10th UK Particle Technology Forum, 2009 Jul 1-2; Birmingham, UK (won 2nd place in poster competition).
11) Chellappah K Tarleton ES and Wakeman RJ (2010). The porosity, permeability and restructuring of heterogeneous filter cakes, Chemical Engineering and Technology (In Press).
12) Coackley P and Jones RS (1956). Vacuum sludge filtration, Sewage and industrial waste journal.
13) Coprapciogh YM (1981). Formation of filter cakes, Filtration and separation, Vol. 18, No. 4, pp. 324-326.
14) Eberl M, Landman KA and Scales PJ (1996). Scale-up procedures and test methods in filtration: A test case on kaolin plant data (vol 103, pg 1, 1995). Colloids and Surfaces A Physicochem. Eng. Aspects, 111, 159-159.
15) Gale RS (1975). Comment on dimensional equation, Effluent and water treatment journal, p 422.

16) Grace HP (1953). Resistance and Compressibility of Filter Cakes, Part II: Under Conditions of Pressure Filtration, Chemical Engineering Progress, Vol. 49, No. 7, pp. 367-377.
17) Hemant RM (1981). Cakes filtration empirically incorporating parcile migration, Filtration and separation. Vol 18, No. 1,pp. 20-24.
18) Lee DJ and Wang CH (2000). Theories of cake filtration and consolidation and implications to sludge dewatering, Water Research, vol. 34(1), pp. 1-20
19) Svarovsky L(2000). Solid-Liquid Separation, 4th Ed., Butterworth-Heinemann, Oxford, UK, pp. 1-29, 303-334, 335-348, 368-392.
20) Terzaghi K (1966). Theoretical soil mechanics. John Wiley and Sons, Inc. N. Y.4th Edition.
21) Tien C (2002). Cake filtration research- A personal view, Powder Technology 127, 1-8.